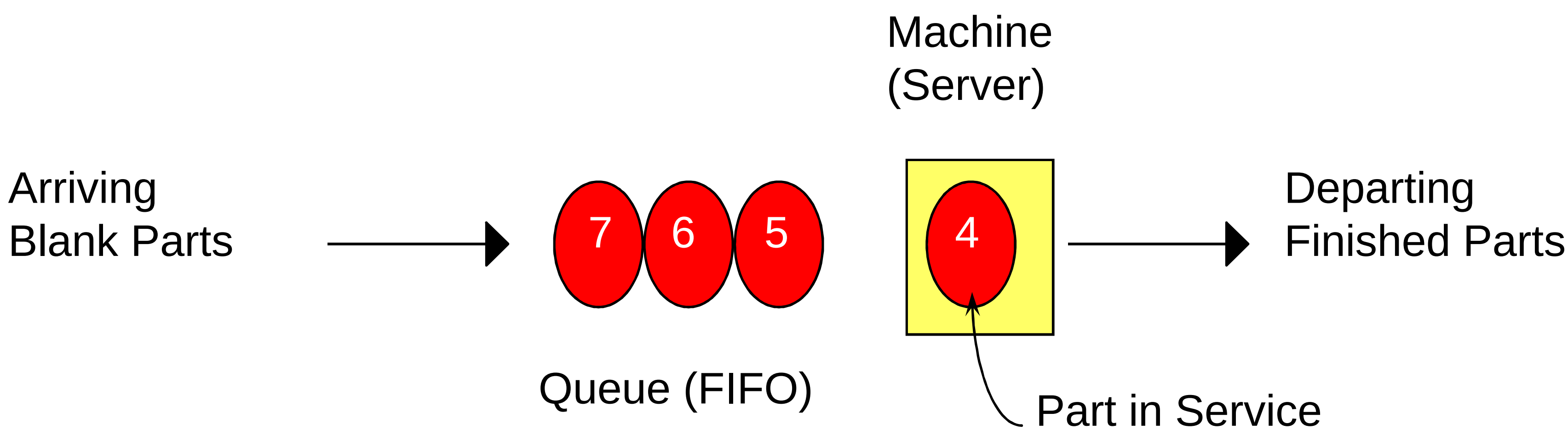
Simulation Concepts and by Hand Simuation

Contents

- **Underlying ideas, methods, and issues in simulation**
- **Software-independent (setting up for Arena)**
- **Centered around an example of a simple processing system**
  - Decompose the problem
  - Terminology
  - Simulation by hand
  - Some basic statistical issues
  - Overview of a simulation study

The System:
A Simple Processing System



The System Contd…

- **General intent:**
  - Estimate expected production
  - Waiting time in queue, queue length, proportion of time machine is busy
- **Time units**
  - Can use different units in different places … must declare
  - Be careful to check the units when specifying inputs
  - Declare *base time units* for internal calculations, outputs
  - Be reasonable (interpretation, round-off error)

Model Specifics

- **Initially (time 0) empty and idle**
- **Base time units: minutes**
- **Input data (assume given for now …), in minutes:**

Part Number	Arrival Time	Interarrival Time	Service Time
1	0.00	1.73	2.90
2	1.73	1.35	1.76
3	3.08	0.71	3.39
4	3.79	0.62	4.52
5	4.41	14.28	4.46
6	18.69	0.70	4.36
7	19.39	15.52	2.07
8	34.91	3.15	3.36
9	38.06	1.76	2.37
10	39.82	1.00	5.38
11	40.82	.	.
.	.	.	.
.	.	.	.

- **Stop when 20 minutes of (simulated) time have passed**

Goals of the Study: Output Performance Measures

- *Total production* of parts over the run (*P*)
- *Average waiting time* of parts in queue:

$$\frac{\sum_{i=1}^{N} WQ_i}{N}$$

N = no. of parts completing queue wait
WQ_i = waiting time in queue of ith part

- *Maximum waiting time* of parts in queue:

$$\max_{i=1,\ldots,N} WQ_i$$

Contd…

- *Time-average number of parts in queue*:

$$\frac{\int_0^{20} Q(t)\,dt}{20}$$

$Q(t)$ = number of parts in queue at time t

- *Maximum number of parts in queue*: $\max_{0 \le t \le 20} Q(t)$
- *Average* and *maximum total time in system* of parts (a.k.a. *cycle time*):

$$\frac{\sum_{i=1}^{P} TS_i}{P}, \quad \max_{i=1,\dots,P} TS_i$$

TS_i = time in system of part i

Contd…

- *Utilization* of the machine (proportion of time busy)

$$\frac{\int_0^{20} B(t)\,dt}{20}, \quad B(t) = \begin{cases} 1 & \text{if the machine is busy at time } t \\ 0 & \text{if the machine is idle at time } t \end{cases}$$

- Many others possible (information overload?)

Analysis Options

- **Educated guessing**
  - Average interarrival time = 4.08 minutes
  - Average service time = 3.46 minutes
  - So (on average) parts are being processed faster than they arrive
    - System has a chance of operating in a stable way in the long run, i.e., might not “explode”
    - If all interarrivals and service times were exactly at their mean, there would never be a queue
    - But the data clearly exhibit variability, so a queue could form
  - If we’d had average interarrival < average service time, and this persisted, then queue would explode
  - Guessing has its limits …

Analysis Options (cont'd.)

- Queuing theory
  - Requires additional assumptions about the model
  - Popular, simple model: *M/M/1 queue*
    - Interarrival times ~ exponential
    - Service times ~ exponential, independent of interarrivals
    - Must have E(service) < E(interarrival)
    - Steady-state (long-run, forever)
    - Exact analytic results; e.g., average waiting time in queue is

$$\frac{«_S^2}{«_A - «_S}, \qquad \begin{aligned} «_A &= \text{E(interarrival time)} \\ «_S &= \text{E(service time)} \end{aligned}$$

  - Problems: validity, estimating means, time frame
  - Often useful as first-cut approximation

Mechanistic Simulation

- **Individual operations** (arrivals, service times) will occur exactly as in reality
- **Movements, changes occur at the right** “time,” in the right order
- **Different pieces interact**
- **Install “observers”** to get output performance measures
- **Concrete, “brute-force” analysis approach**
- **Nothing mysterious or subtle**
  - But a lot of details, bookkeeping
  - Simulation software keeps track of things for you

Pieces of a Simulation Model

- ***Entities***
  - "Players" that move around, change status, affect and are affected by other entities
  - ***Dynamic objects*** — get created, move around, leave (maybe)
  - Usually represent "real" things
    - Our model: entities are the parts
  - Can have "fake" entities for modeling "tricks"
    - Breakdown demon
  - Usually have multiple ***realizations*** floating around
  - Can have **different types of entities** concurrently
  - Usually, identifying the types of entities is the first thing to do in building a model

Pieces of a Simulation Model (cont'd.)

- ***Attributes***
  - Characteristic of all entities: describe, differentiate
  - All entities have same attribute "slots" but different values for different entities, for example:
    - Time of arrival
    - Due date
    - Priority
    - Color
  - Attribute value tied to a specific entity
  - Like "local" (to entities) variables
  - Some automatic in Arena, some you define

Pieces of a Simulation Model (cont'd.)

- (Global) ***Variables***
  - Reflects a **characteristic of the whole model**, not of specific entities
  - Used for many different kinds of things
    - Travel time between all station pairs
    - Number of parts in system
    - Simulation clock (built-in Arena variable)
  - Name, value of which there's only one copy for the whole model
  - Not tied to entities
  - Entities can access, change variables
  - Writing on the wall
  - Some built-in by Arena, you can define others

Pieces of a Simulation Model (cont'd.)

- ***Resources***
  - What entities compete for
    - People
    - Equipment
    - Space
  - Entity ***seizes*** a resource, uses it, ***releases*** it
  - Think of a ***resource being assigned to an entity***, rather than an entity "belonging to" a resource
  - "A" resource can have several ***units*** of capacity
    - Seats at a table in a restaurant
    - Identical ticketing agents at an airline counter
  - ***Number of units*** of resource can be changed during the simulation

Pieces of a Simulation Model (cont'd.)

- ***Queues***
  - Place for entities to wait when they can't move on (maybe since the resource they want to seize is not available)
  - Have names, often tied to a corresponding resource
  - Can have a finite capacity to model limited space — have to model what to do if an entity shows up to a queue that's already full
  - Usually watch the ***length of a queue, waiting time in it***

Pieces of a Simulation Model (cont'd.)

- ***Statistical accumulators***
  - ***Variables*** that "watch" what's happening
  - Depend on ***output performance measures*** desired
  - "Passive" in model — don't participate, just watch
  - Many are automatic in Arena, but *some you may have to set up and maintain during the simulation*
  - At end of simulation, used to *compute final output performance measures*

Pieces of a Simulation Model (cont'd.)

- Statistical accumulators for the simple processing system
  - Number of parts produced so far
  - Total of the waiting times spent in queue so far
  - No. of parts that have gone through the queue
  - Max time in queue we've seen so far
  - Total of times spent in system
  - Max time in system we've seen so far
  - Area so far under queue-length curve $Q(t)$
  - Max of $Q(t)$ so far
  - Area so far under server-busy curve $B(t)$

Simulation Dynamics: The Event-Scheduling

- Identify characteristic *events*
- Decide on *logic* for each type of event to
  - Effect *state changes* for each event type
  - Observe statistics
  - Update times of future events (maybe of this type, other types)
- Keep a simulation *clock*, future *event calendar*
- *Jump* from one event to the next, process, observe statistics, update event calendar
- Must specify an appropriate *stopping rule*
- Usually done with general-purpose programming language (C, FORTRAN, etc.)

Events for the Simple Processing System

- ***Arrival*** of a new part to the system
  - Update time-persistent statistical accumulators (from last event to now)
    - Area under $Q(t)$
    - Max of $Q(t)$
    - Area under $B(t)$
  - "Mark" arriving part with current time (use later)
  - If machine is idle:
    - Start processing (schedule departure), Make machine busy, Tally waiting time in queue (0)
  - Else (machine is busy):
    - Put part at end of queue, increase queue-length variable
  - Schedule the next arrival event

Events for the Simple Processing System (cont'd.)

- ***Departure*** (when a service is completed)
  - Increment number-produced state accumulator
  - Compute & tally time in system (now - time of arrival)
  - Update time-persistent statistics (as in arrival event)
  - If queue is non-empty:
    - Take first part out of queue, compute & tally its waiting time in queue, begin service (schedule departure event)
  - Else (queue is empty):
    - Make the machine idle (Note: there will be no departure event scheduled on the future events calendar, which is as desired)

Events for the Simple Processing System (cont'd.)

- ***The End***
  - Update time-persistent statistics (to end of the simulation)
  - Compute final output performance measures using current (= final) values of statistical accumulators
- After each event, the event calendar's top record is removed to see what time it is, what to do
- Also must initialize everything

Some Additional Specifics for the Simple Processing System

- Simulation clock variable (internal in Arena)
- **Event calendar**: List of event *records*:
  - [Entity No., Event Time, Event Type]
  - Keep *ranked* in increasing order on Event Time
  - Next event always in top record
  - Initially, schedule first Arrival, The End (Dep.?)
- **State variables**: describe current status
  - Server status $B(t)$ = 1 for busy, 0 for idle
  - Number of customers in queue $Q(t)$
  - Times of arrival of each customer now in queue (a list of random length)

Simulation by Hand

- Manually track ***state variables***, ***statistical accumulators***
- Use "given" interarrival, service times
- Keep track of event calendar
- "Lurch" clock from one event to the next

Model Specifics

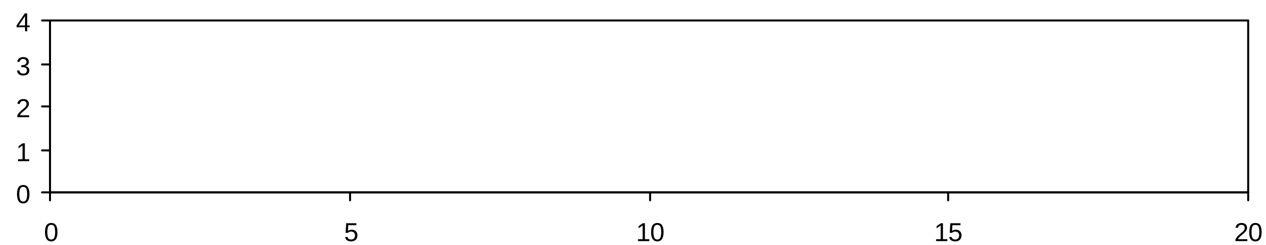
- **Initially (time 0) empty and idle**
- **Base time units: minutes**
- **Input data (assume given for now …), in minutes:**

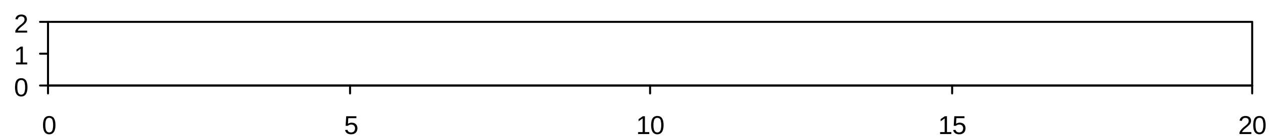
Part Number	Arrival Time	Interarrival Time	Service Time
1	0.00	1.73	2.90
2	1.73	1.35	1.76
3	3.08	0.71	3.39
4	3.79	0.62	4.52
5	4.41	14.28	4.46
6	18.69	0.70	4.36
7	19.39	15.52	2.07
8	34.91	3.15	3.36
9	38.06	1.76	2.37
10	39.82	1.00	5.38
11	40.82	.	.
.	.	.	.
.	.	.	.

- **Stop when 20 minutes of (simulated) time have passed**

Simulation by Hand: Setup

<table>
<tr><td>System</td><td>Clock</td><td>B(t)</td><td>Q(t)</td><td>Arrival times of custs. in queue</td><td>Event calendar</td></tr>
<tr><td>Number of completed waiting times in queue</td><td colspan="3">Total of waiting times in queue</td><td>Area under Q(t)</td><td>Area under B(t)</td></tr>
<tr><td>Q(t) graph

B(t) graph</td><td colspan="5">



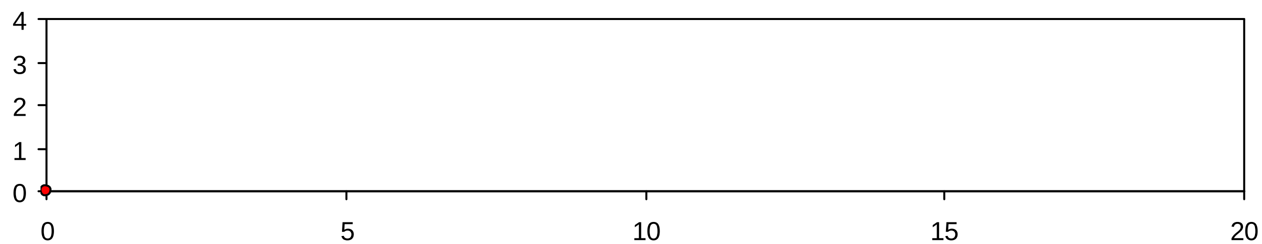
Time (Minutes)</td></tr>
<tr><td>Interarrival times</td><td colspan="5">1.73, 1.35, 0.71, 0.62, 14.28, 0.70, 15.52, 3.15, 1.76, 1.00, ...</td></tr>
<tr><td>Service times</td><td colspan="5">2.90, 1.76, 3.39, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38, ...</td></tr>
</table>

Simulation by Hand:
t = 0.00, Initialize

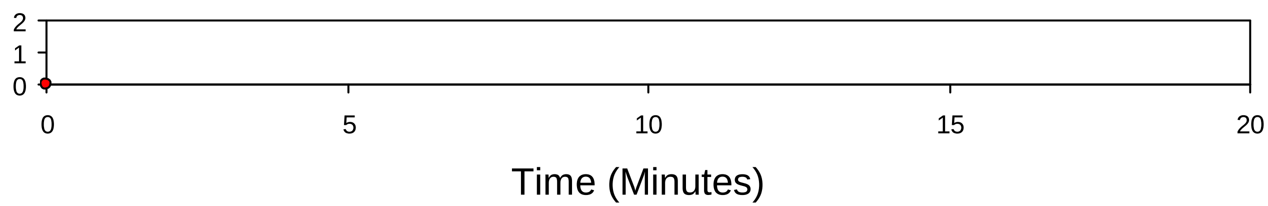
System	Clock	$B(t)$	$Q(t)$	Arrival times of custs. in queue	Event calendar
	0.00	0	0	<empty>	[1, 0.00, Arr] [–, 20.00, End]

Number of completed waiting times in queue	Total of waiting times in queue	Area under $Q(t)$	Area under $B(t)$
0	0.00	0.00	0.00

$Q(t)$ graph

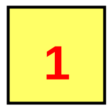


$B(t)$ graph



Interarrival times	1.73, 1.35, 0.71, 0.62, 14.28, 0.70, 15.52, 3.15, 1.76, 1.00, ...
Service times	2.90, 1.76, 3.39, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38, ...

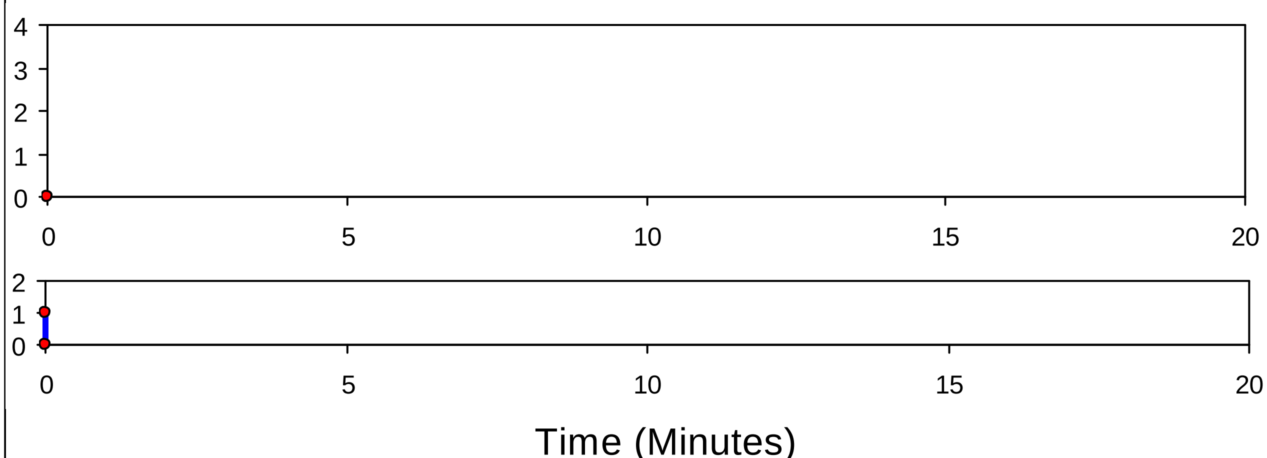
Simulation by Hand:
t = 0.00, Arrival of Part 1

System	Clock	$B(t)$	$Q(t)$	Arrival times of custs. in queue	Event calendar
	0.00	1	0	<empty>	[2, 1.73, Arr] [1, 2.90, Dep] [–, 20.00, End]

Number of completed waiting times in queue	Total of waiting times in queue	Area under $Q(t)$	Area under $B(t)$
1	0.00	0.00	0.00

$Q(t)$ graph

$B(t)$ graph



Interarrival times	~~1.73~~, 1.35, 0.71, 0.62, 14.28, 0.70, 15.52, 3.15, 1.76, 1.00, ...
Service times	~~2.90~~, 1.76, 3.39, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38, ...

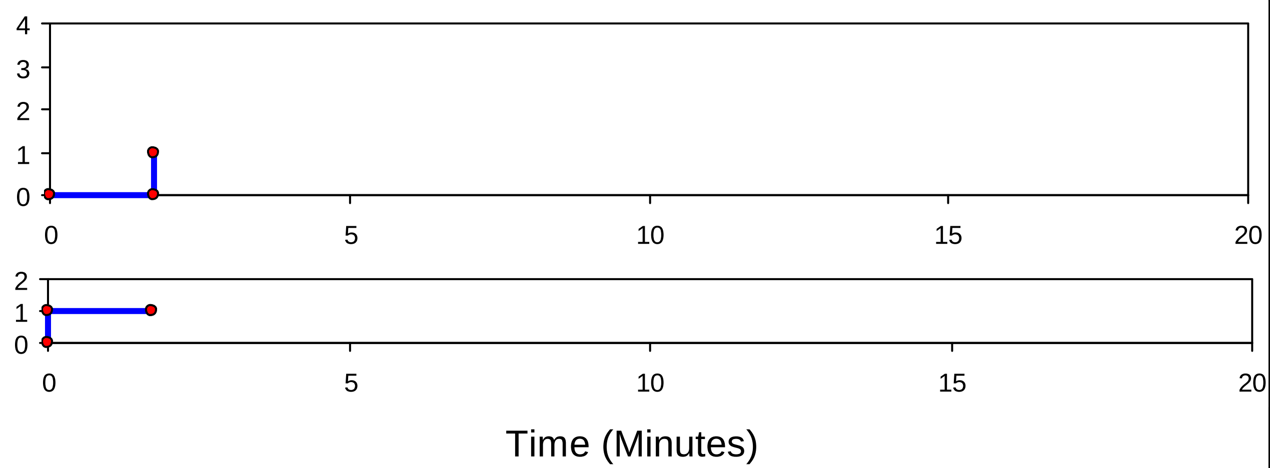
Simulation by Hand:
t = 1.73, Arrival of Part 2

System	Clock	*B*(*t*)	*Q*(*t*)	Arrival times of custs. in queue	Event calendar
2 [1]	1.73	1	1	(1.73)	[1, 2.90, Dep] [3, 3.08, Arr] [–, 20.00, End]

Number of completed waiting times in queue	Total of waiting times in queue	Area under *Q*(*t*)	Area under *B*(*t*)
1	0.00	0.00	1.73

Q(*t*) graph

B(*t*) graph



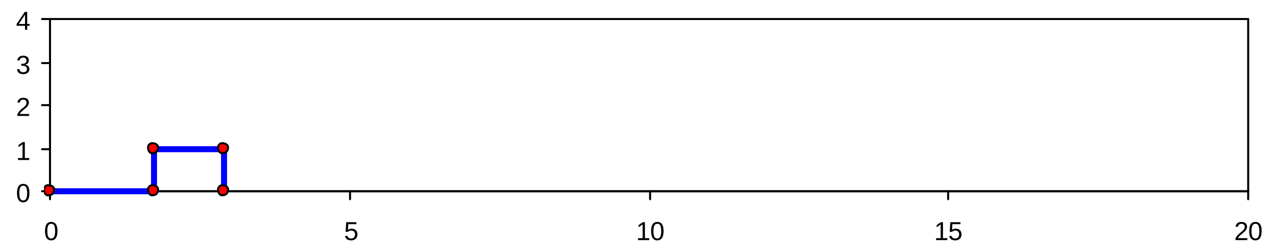
Interarrival times	~~1.73~~, ~~1.35~~, 0.71, 0.62, 14.28, 0.70, 15.52, 3.15, 1.76, 1.00, ...
Service times	~~2.90~~, 1.76, 3.39, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38, ...

Simulation by Hand:
t = 2.90, Departure of Part 1

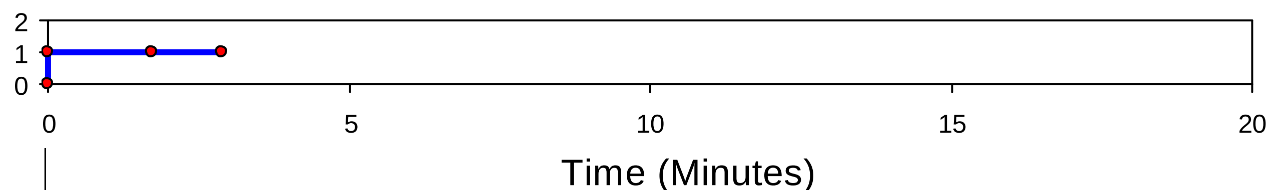
System	Clock	$B(t)$	$Q(t)$	Arrival times of custs. in queue	Event calendar
2	2.90	1	0	<empty>	[3, 3.08, Arr] [2, 4.66, Dep] [–, 20.00, End]

Number of completed waiting times in queue	Total of waiting times in queue	Area under $Q(t)$	Area under $B(t)$
2	1.17	1.17	2.90

$Q(t)$ graph



$B(t)$ graph



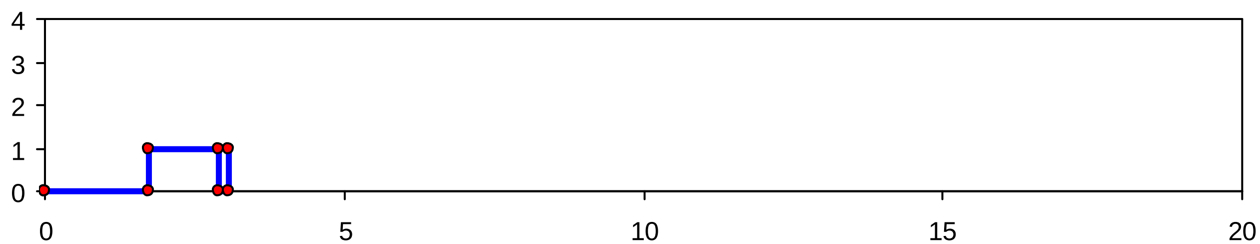
Interarrival times	~~1.73~~, ~~1.35~~, 0.71, 0.62, 14.28, 0.70, 15.52, 3.15, 1.76, 1.00, ...
Service times	~~2.90~~, ~~1.76~~, 3.39, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38, ...

Simulation by Hand:
t = 3.08, Arrival of Part 3

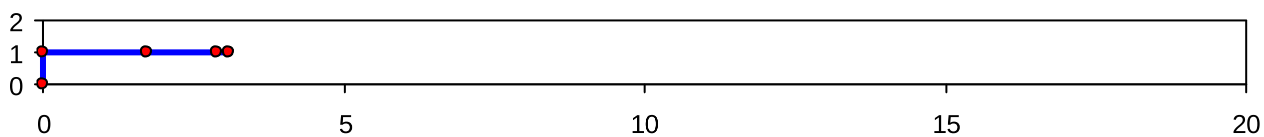
System	Clock	*B*(*t*)	*Q*(*t*)	Arrival times of custs. in queue	Event calendar
3 [2]	3.08	1	1	(3.08)	[4, 3.79, Arr] [2, 4.66, Dep] [–, 20.00, End]

Number of completed waiting times in queue	Total of waiting times in queue	Area under *Q*(*t*)	Area under *B*(*t*)
2	1.17	1.17	3.08

Q(*t*) graph



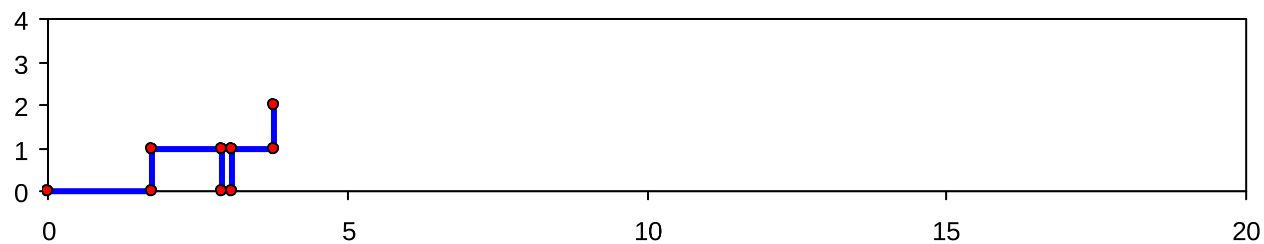
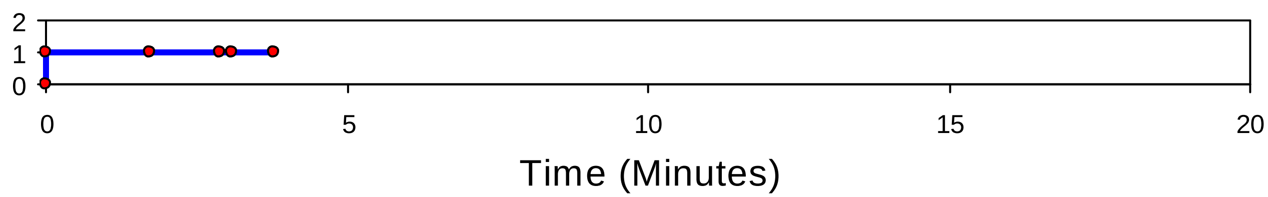
B(*t*) graph



Interarrival times	~~1.73~~, ~~1.35~~, ~~0.71~~, 0.62, 14.28, 0.70, 15.52, 3.15, 1.76, 1.00, ...
Service times	~~2.90~~, ~~1.76~~, 3.39, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38, ...

Simulation by Hand:
t = 3.79, Arrival of Part 4

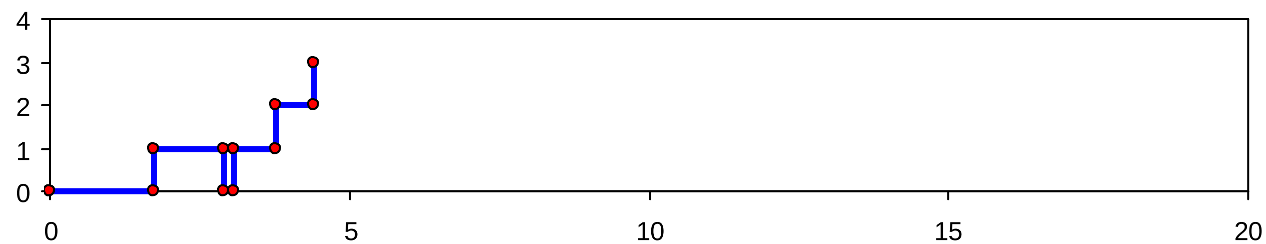
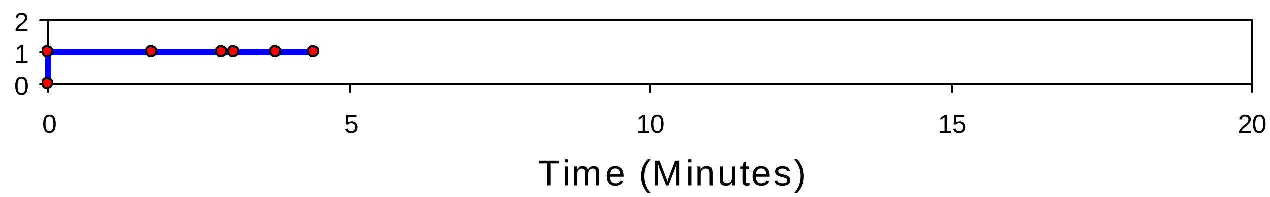
<table>
<tr><td>System
4 3 [2]</td><td>Clock
3.79</td><td>B(t)
1</td><td colspan="2">Q(t)
2</td><td>Arrival times of custs. in queue
(3.79, 3.08)</td><td>Event calendar
[5, 4.41, Arr]
[2, 4.66, Dep]
[–, 20.00, End]</td></tr>
<tr><td>Number of completed waiting times in queue
2</td><td colspan="3">Total of waiting times in queue
1.17</td><td colspan="2">Area under Q(t)
1.88</td><td>Area under B(t)
3.79</td></tr>
<tr><td>Q(t) graph

B(t) graph</td><td colspan="6"></td></tr>
<tr><td>Interarrival times</td><td colspan="6">~~1.73~~, ~~1.35~~, ~~0.71~~, ~~0.62~~, 14.28, 0.70, 15.52, 3.15, 1.76, 1.00, ...</td></tr>
<tr><td>Service times</td><td colspan="6">~~2.90~~, ~~1.76~~, 3.39, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38, ...</td></tr>
</table>

Simulation by Hand:
t = 4.41, Arrival of Part 5

System	Clock	*B*(*t*)	*Q*(*t*)	Arrival times of custs. in queue	Event calendar
5 4 3 **2**	4.41	1	3	(4.41, 3.79, 3.08)	[2, 4.66, Dep] [6, 18.69, Arr] [–, 20.00, End]

Number of completed waiting times in queue	Total of waiting times in queue	Area under *Q*(*t*)	Area under *B*(*t*)
2	1.17	3.12	4.41

Q(*t*) graph	
B(*t*) graph	
Interarrival times	~~1.73~~, ~~1.35~~, ~~0.71~~, ~~0.62~~, ~~14.28~~, 0.70, 15.52, 3.15, 1.76, 1.00, ...
Service times	~~2.90~~, ~~1.76~~, 3.39, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38, ...

Simulation by Hand:
t = 4.66, Departure of Part 2

<table>
<tr><td>System

5 4 [3]</td><td>Clock

4.66</td><td>B(t)

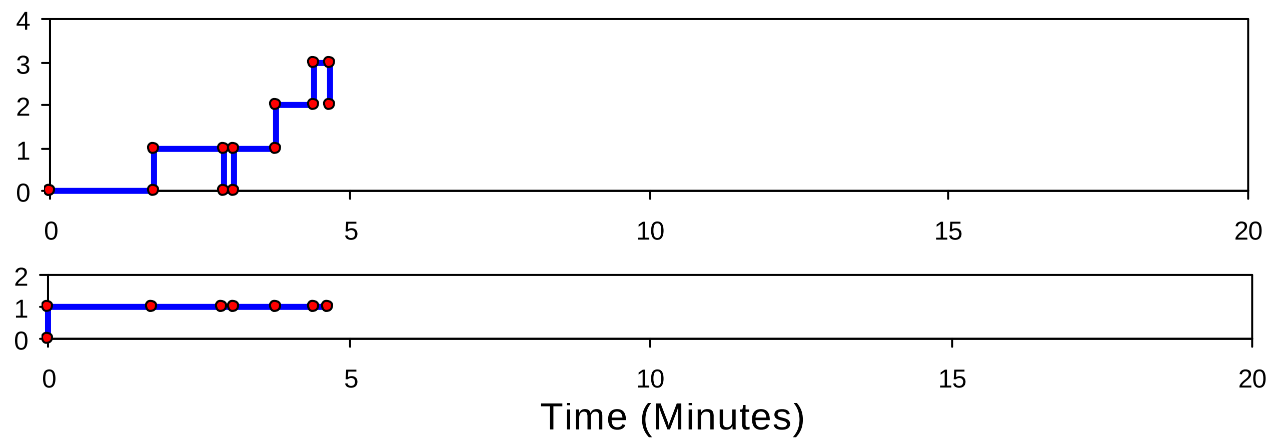
1</td><td>Q(t)

2</td><td>Arrival times of custs. in queue
(4.41, 3.79)</td><td>Event calendar
[3, 8.05, Dep]
[6, 18.69, Arr]
[–, 20.00, End]</td></tr>
<tr><td>Number of completed waiting times in queue
3</td><td colspan="2">Total of waiting times in queue

2.75</td><td colspan="2">Area under Q(t)

3.87</td><td>Area under B(t)

4.66</td></tr>
<tr><td>Q(t) graph

B(t) graph</td><td colspan="5">
</td></tr>
<tr><td>Interarrival times</td><td colspan="5">~~1.73~~, ~~1.35~~, ~~0.71~~, ~~0.62~~, ~~14.28~~, 0.70, 15.52, 3.15, 1.76, 1.00, ...</td></tr>
<tr><td>Service times</td><td colspan="5">~~2.90~~, ~~1.76~~, ~~3.39~~, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38, ...</td></tr>
</table>

Simulation by Hand:
t = 8.05, Departure of Part 3

<table>
<tr><td>System
5 | 4</td><td>Clock

8.05</td><td>$B(t)$

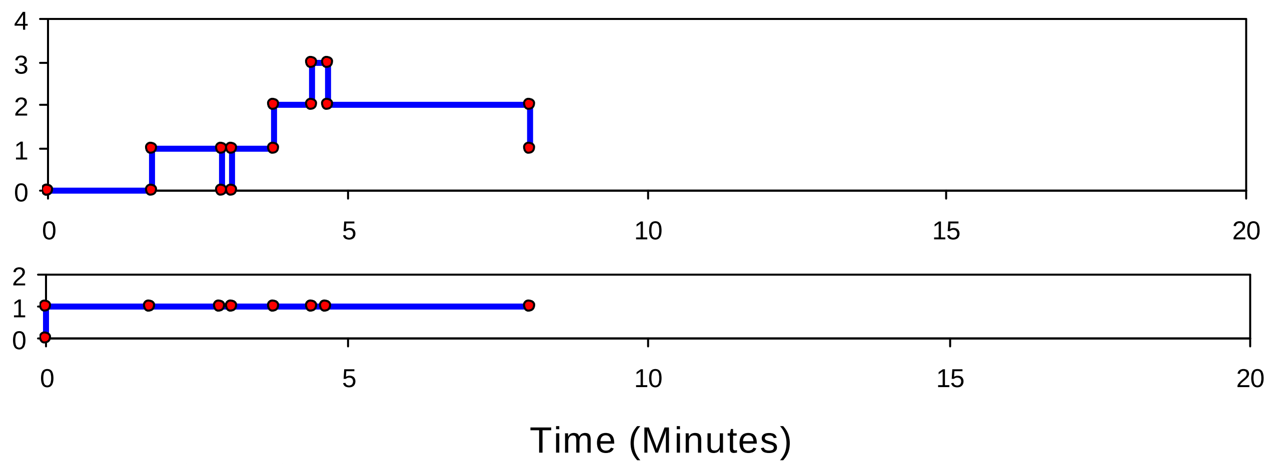
1</td><td>$Q(t)$

1</td><td colspan="2">Arrival times of custs. in queue
(4.41)</td><td>Event calendar
[4, 12.57, Dep]
[6, 18.69, Arr]
[–, 20.00, End]</td></tr>
<tr><td>Number of completed waiting times in queue
4</td><td colspan="3">Total of waiting times in queue

7.01</td><td colspan="2">Area under $Q(t)$

10.65</td><td>Area under $B(t)$

8.05</td></tr>
<tr><td>$Q(t)$ graph

$B(t)$ graph</td><td colspan="6"></td></tr>
<tr><td>Interarrival times</td><td colspan="6">~~1.73~~, ~~1.35~~, ~~0.71~~, ~~0.62~~, ~~14.28~~, 0.70, 15.52, 3.15, 1.76, 1.00, ...</td></tr>
<tr><td>Service times</td><td colspan="6">~~2.90~~, ~~1.76~~, ~~3.39~~, ~~4.52~~, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38, ...</td></tr>
</table>

Simulation by Hand:
t = 12.57, Departure of Part 4

<table>
<tr><td>System

5</td><td>Clock

12.57</td><td>B(t)

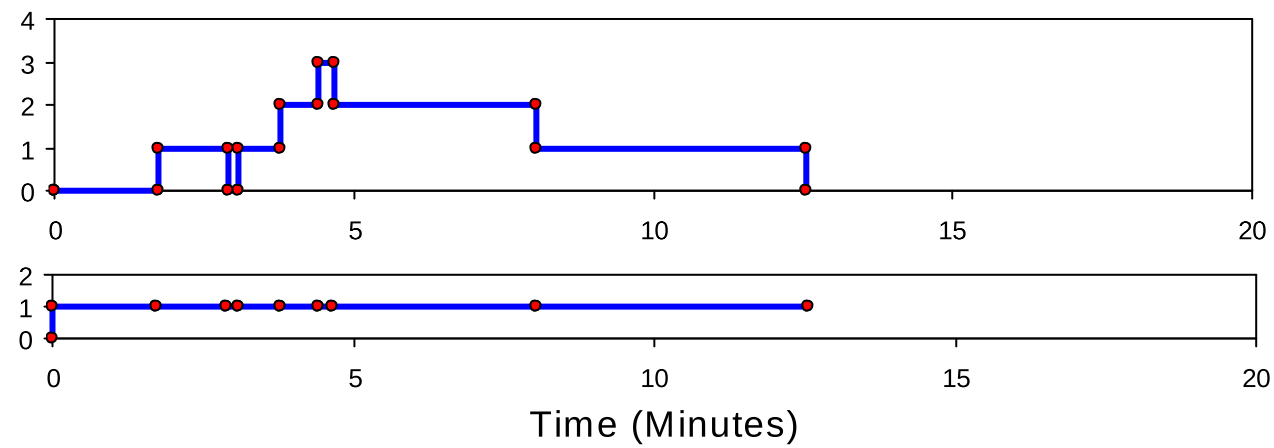
1</td><td>Q(t)

0</td><td>Arrival times of custs. in queue
()</td><td colspan="2">Event calendar
[5, 17.03, Dep]
[6, 18.69, Arr]
[–, 20.00, End]</td></tr>
<tr><td>Number of completed waiting times in queue
5</td><td colspan="2">Total of waiting times in queue

15.17</td><td colspan="2">Area under Q(t)

15.17</td><td colspan="2">Area under B(t)

12.57</td></tr>
<tr><td>Q(t) graph

B(t) graph</td><td colspan="6"></td></tr>
<tr><td>Interarrival times</td><td colspan="6">~~1.73~~, ~~1.35~~, ~~0.71~~, ~~0.62~~, ~~14.28~~, 0.70, 15.52, 3.15, 1.76, 1.00, ...</td></tr>
<tr><td>Service times</td><td colspan="6">~~2.90~~, ~~1.76~~, ~~3.39~~, ~~4.52~~, ~~4.46~~, 4.36, 2.07, 3.36, 2.37, 5.38, ...</td></tr>
</table>

Simulation by Hand:

t = 17.03, Departure of Part 5

<table>
<tr><td>System</td><td>Clock

17.03</td><td>B(t)

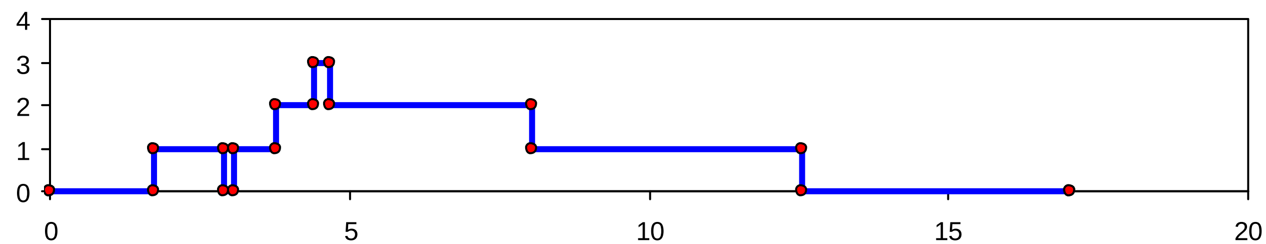
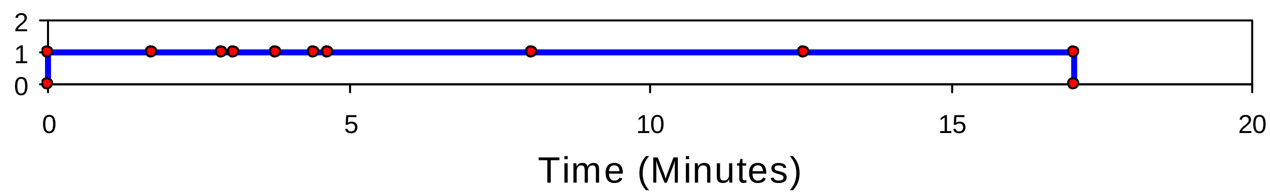
0</td><td>Q(t)

0</td><td>Arrival times of custs. in queue
()</td><td>Event calendar
[6, 18.69, Arr]
[–, 20.00, End]</td></tr>
<tr><td>Number of completed waiting times in queue
5</td><td colspan="2">Total of waiting times in queue

15.17</td><td colspan="2">Area under Q(t)

15.17</td><td>Area under B(t)

17.03</td></tr>
<tr><td>Q(t) graph

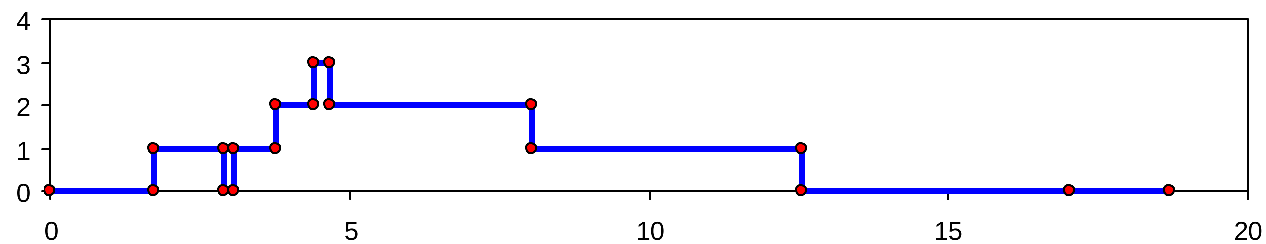
B(t) graph</td><td colspan="5"></td></tr>
<tr><td>Interarrival times</td><td colspan="5">~~1.73~~, ~~1.35~~, ~~0.71~~, ~~0.62~~, ~~14.28~~, 0.70, 15.52, 3.15, 1.76, 1.00, ...</td></tr>
<tr><td>Service times</td><td colspan="5">~~2.90~~, ~~1.76~~, ~~3.39~~, ~~4.52~~, ~~4.46~~, 4.36, 2.07, 3.36, 2.37, 5.38, ...</td></tr>
</table>

Simulation by Hand:
t = 18.69, Arrival of Part 6

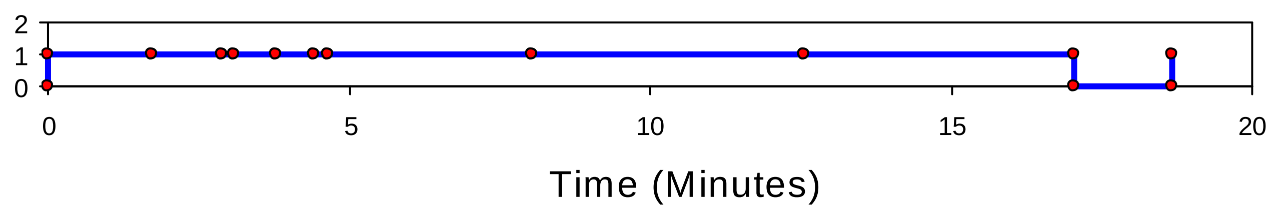
System	Clock	$B(t)$	$Q(t)$	Arrival times of custs. in queue	Event calendar
6	18.69	1	0	()	[7, 19.39, Arr] [–, 20.00, End] [6, 23.05, Dep]

Number of completed waiting times in queue	Total of waiting times in queue	Area under $Q(t)$	Area under $B(t)$
6	15.17	15.17	17.03

$Q(t)$ graph



$B(t)$ graph



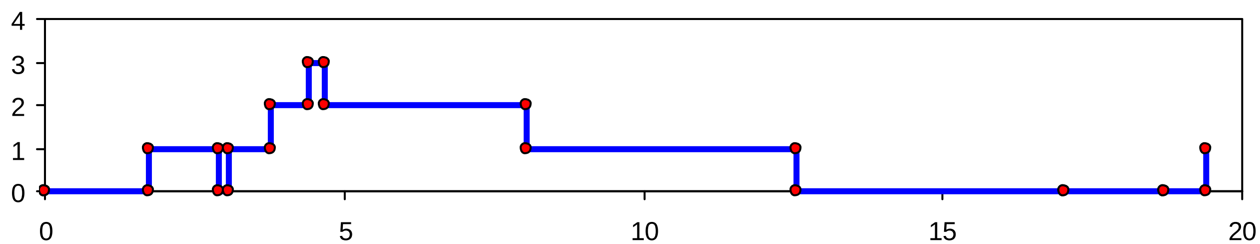
Interarrival times	1.73, 1.35, 0.71, 0.62, 14.28, 0.70, 15.52, 3.15, 1.76, 1.00, ...
Service times	2.90, 1.76, 3.39, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38, ...

Simulation by Hand:
t = 19.39, Arrival of Part 7

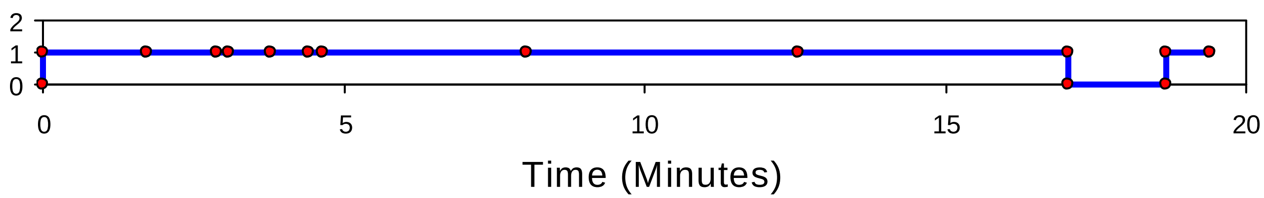
System	Clock	*B*(*t*)	*Q*(*t*)	Arrival times of custs. in queue	Event calendar
7 6	19.39	1	1	(19.39)	[–, 20.00, End] [6, 23.05, Dep] [8, 34.91, Arr]

Number of completed waiting times in queue	Total of waiting times in queue	Area under *Q*(*t*)	Area under *B*(*t*)
6	15.17	15.17	17.73

Q(*t*) graph

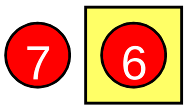


B(*t*) graph



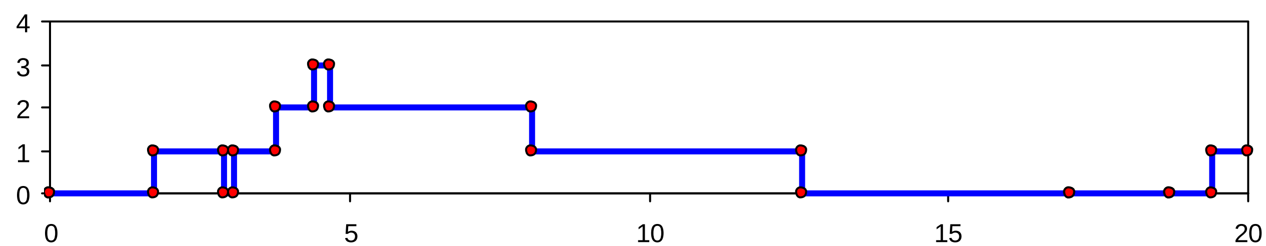
Interarrival times	~~1.73~~, ~~1.35~~, ~~0.71~~, ~~0.62~~, ~~14.28~~, ~~0.70~~, ~~15.52~~, 3.15, 1.76, 1.00, ...
Service times	~~2.90~~, ~~1.76~~, ~~3.39~~, ~~4.52~~, ~~4.46~~, ~~4.36~~, 2.07, 3.36, 2.37, 5.38, ...

Simulation by Hand:
t = 20.00, The End

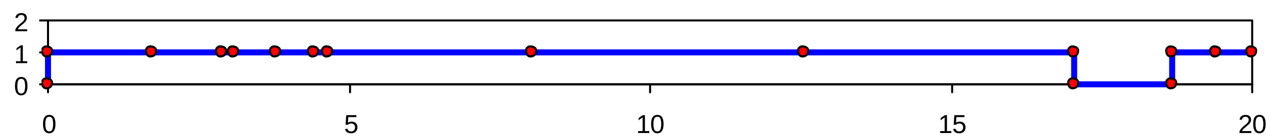
System	Clock	$B(t)$	$Q(t)$	Arrival times of custs. in queue	Event calendar
	20.00	1	1	(19.39)	[6, 23.05, Dep] [8, 34.91, Arr]

Number of completed waiting times in queue	Total of waiting times in queue	Area under $Q(t)$	Area under $B(t)$
6	15.17	15.78	18.34

$Q(t)$ graph



$B(t)$ graph



Interarrival times	~~1.73~~, ~~1.35~~, ~~0.71~~, ~~0.62~~, ~~14.28~~, ~~0.70~~, ~~15.52~~, 3.15, 1.76, 1.00, ...
Service times	~~2.90~~, ~~1.76~~, ~~3.39~~, ~~4.52~~, ~~4.46~~, ~~4.36~~, 2.07, 3.36, 2.37, 5.38, ...

Simulation by Hand: Finishing Up

- Average waiting time in queue:

$$\frac{\text{Total of times in queue}}{\text{No. of times in queue}} = \frac{15.17}{6} = 2.53 \text{ minutes per part}$$

- Time-average number in queue:

$$\frac{\text{Area under } Q(t) \text{ curve}}{\text{Final clock value}} = \frac{15.78}{20} = 0.79 \text{ part}$$

- Utilization of drill press:

$$\frac{\text{Area under } B(t) \text{ curve}}{\text{Final clock value}} = \frac{18.34}{20} = 0.92 \text{ (dimensionless)}$$

Complete Record of the Hand Simulation

Just-Finished Event			Variables		Attributes		Statistical Accumulators									Event Calendar
Entity No.	Time t	Event Type	$Q(t)$	$B(t)$	Arrival Times: (In Queue)	In Service	P	N	ΣWQ	WQ^*	ΣTS	TS^*	$\int Q$	Q^*	$\int B$	[Entity No., Time, Type]
–	0.00	Init	0	0	( )	–	0	0	0.00	0.00	0.00	0.00	0.00	0	0.00	[1, 0.00, Arr] [–, 20.00, End]
1	0.00	Arr	0	1	( )	0.00	0	1	0.00	0.00	0.00	0.00	0.00	0	0.00	[2, 1.73, Arr] [1, 2.90, Dep] [–, 20.00, End]
2	1.73	Arr	1	1	(1.73)	0.00	0	1	0.00	0.00	0.00	0.00	0.00	1	1.73	[1, 2.90, Dep] [3, 3.08, Arr] [–, 20.00, End]
1	2.90	Dep	0	1	( )	1.73	1	2	1.17	1.17	2.90	2.90	1.17	1	2.90	[3, 3.08, Arr] [2, 4.66, Dep] [–, 20.00, End]
3	3.08	Arr	1	1	(3.08)	1.73	1	2	1.17	1.17	2.90	2.90	1.17	1	3.08	[4, 3.79, Arr] [2, 4.66, Dep] [–, 20.00, End]
4	3.79	Arr	2	1	(3.79, 3.08)	1.73	1	2	1.17	1.17	2.90	2.90	1.88	2	3.79	[5, 4.41, Arr] [2, 4.66, Dep] [–, 20.00, End]
5	4.41	Arr	3	1	(4.41, 3.79, 3.08)	1.73	1	2	1.17	1.17	2.90	2.90	3.12	3	4.41	[2, 4.66, Dep] [6, 18.69, Arr] [–, 20.00, End]
2	4.66	Dep	2	1	(4.41, 3.79)	3.08	2	3	2.75	1.58	5.83	2.93	3.87	3	4.66	[3, 8.05, Dep] [6, 18.69, Arr] [–, 20.00, End]
3	8.05	Dep	1	1	(4.41)	3.79	3	4	7.01	4.26	10.80	4.97	10.65	3	8.05	[4, 12.57, Dep] [6, 18.69, Arr] [–, 20.00, End]
4	12.57	Dep	0	1	( )	4.41	4	5	15.17	8.16	19.58	8.78	15.17	3	12.57	[5, 17.03, Dep] [6, 18.69, Arr] [–, 20.00, End]
5	17.03	Dep	0	0	( )	–	5	5	15.17	8.16	32.20	12.62	15.17	3	17.03	[6, 18.69, Arr] [–, 20.00, End]
6	18.69	Arr	0	1	( )	18.69	5	6	15.17	8.16	32.20	12.62	15.17	3	17.03	[7, 19.39, Arr] [–, 20.00, End] [6, 23.05, Dep]
7	19.39	Arr	1	1	(19.39)	18.69	5	6	15.17	8.16	32.20	12.62	15.17	3	17.73	[–, 20.00, End] [6, 23.05, Dep] [8, 34.91, Arr]
–	20.00	End	1	1	(19.39)	18.69	5	6	15.17	8.16	32.20	12.62	15.78	3	18.34	[6, 23.05, Dep] [8, 34.91, Arr]

Event-Scheduling Logic via Programming

- Clearly well suited to standard programming
- Often use “utility” libraries for:
  - List processing
  - Random-number generation
  - Random-variant generation
  - Statistics collection
  - Event-list and clock management
  - Summary and output
- Main program ties it together, executes events in order

Simulation Dynamics: The Process-Interaction World View

- Identify characteristic *entities* in the system
- Multiple copies of entities co-exist, interact, compete
- Tell a “story” about what happens to a “typical” entity
- May have many types of entities, “fake” entities for things like machine breakdowns

Randomness in Simulation

- The above was just one "replication" — a sample of size one (not worth much)
- Made a total of five replications:

Performance Measure	Replication 1	2	3	4	5	Sample Avg.	Sample Std. Dev.	95% Half Width
Total production	5	3	6	2	3	3.80	1.64	2.04
Average waiting time in queue	2.53	1.19	1.03	1.62	0.00	1.27	0.92	1.14
Maximum waiting time in queue	8.16	3.56	2.97	3.24	0.00	3.59*	2.93*	3.63*
Average total time in system	6.44	5.10	4.16	6.71	4.26	5.33	1.19	1.48
Maximum total time in system	12.62	6.63	6.27	7.71	4.96	7.64*	2.95*	3.67*
Time-average number of parts in queue	0.79	0.18	0.36	0.16	0.05	0.31	0.29	0.36
Maximum number of parts in queue	3	1	2	1	1	1.60*	0.89*	1.11*
Drill-press utilization	0.92	0.59	0.90	0.51	0.70	0.72	0.18	0.23

Note substantial variability across replications

- Confidence intervals for expected values:

- Confidence intervals for expected values:

In general,

$$\overline{X} \pm t_{n-1,1-\alpha/2}\, s/\sqrt{n}$$

For expected total production,

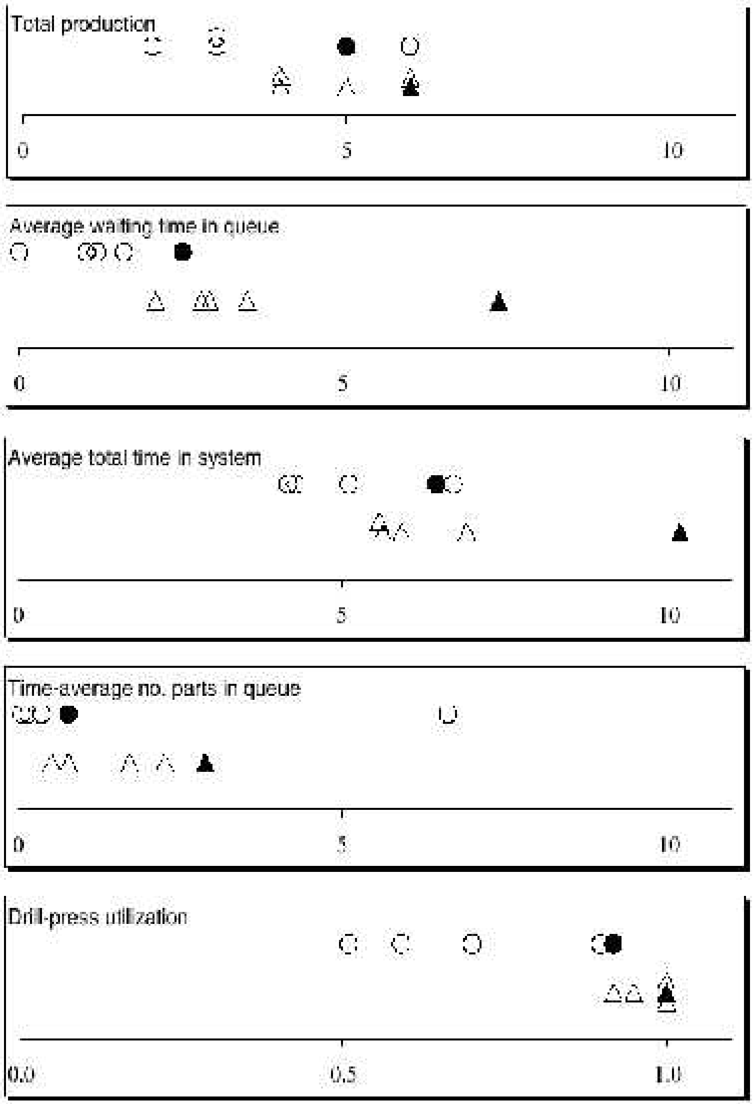
$$3.80 \pm (2.776)(1.64/\sqrt{5})$$

$$3.80 \pm 2.04$$

Comparing Alternatives

- Usually, simulation is used for more than just a single model "configuration"
- Often want to compare alternatives, select or search for the best (via some criterion)
- Simple processing system: What would happen if the arrival rate were to double?
  - Cut interarrival times in half
  - Rerun the model for double-time arrivals
  - Make five replications

Results: Original vs. Double-Time Arrivals



- Original – circles
- Double-time – triangles
- Replication 1 – filled in
- Replications 2-5 – hollow
- Note variability
- Danger of making decisions based on one (first) replication
- Hard to see if there are really differences
- Need: Statistical analysis of simulation output data

Overview of a Simulation Study

- Understand the system
- Be clear about the goals
- Formulate the model representation
- Translate into modeling software
- Verify "program"
- Validate model
- Design experiments
- Make runs
- Analyze, get insight, document results